Dot and Cross Products

Dot Product

Definition 1 $\underline{a}.\underline{b} = a_x b_x + a_y b_y + a_z b_z$

Definition 2 $\underline{a}.\underline{b} = |a||b|\cos\theta$ where θ is the angle between $\underline{a}$ and $\underline{b}$

Definition 3 The dot product of $\underline{a}$ and $\underline{b}$ is the magnitude of one vector times the component of the other vector in the direction of the first vector.

Useful Mathematical Relations:

$\underline{a}.\underline{b} = \underline{b}.\underline{a}$	dot product is commutative
$(\underline{a} + \underline{b}).\underline{c} = \underline{a}.\underline{c} + \underline{b}.\underline{c}$	dot product is distributive
$\underline{a}.\underline{a} = \lvert a \rvert^2$	

Physical Applications:

Work done (W) by a force ($\underline{F}$) undergoing a displacement ($\underline{d}$) is given by $W = \underline{F}.\underline{d}$

Rate of work (R) done by a force ($\underline{F}$) acting on a particle moving with velocity ($\underline{v}$) is $R = \underline{F}.\underline{v}$

Flux of a uniform vector $\underline{E}$ through a surface of area $\underline{A}$ is $\Phi = \underline{E}.\underline{A}$
NB extend this to Gauss's law

If you require a component of a vector quantity ($\underline{a}$) in a certain direction then define a vector ($\underline{n}$) with unit magnitude ($|\underline{n}|$=1) in that direction and then the component required is $\underline{a}.\underline{n}$.

If you know that vectors $\underline{a}$ and $\underline{b}$ are orthogonal then $\underline{a}.\underline{b} = 0$.

Problems

Question 1

a) If a force $\underline{F}$ = (0, 1, 13) N acts on an object which undergoes a displacement $\underline{d}$ = (3, 2, 5) m, what work does the force do?
b) If a force $\underline{F}$ = (3, 1, 7) N acts on an object which undergoes a displacement $\underline{d}$ = (5, 6, -3) m, what work does the force do?
c) If a force $\underline{F}$ = (1, -2, 3) N acts on an object which undergoes a displacement $\underline{d}1$ = (5, 6, -3) m followed by a displacement $\underline{d}2$ = (2, -3, 4) m , what work does the force do over the whole displacement? Calculate this both by calculating the total displacement and also by calculating the work done during each stage of the journey. What property of dot products ensures these are the same?

Question 2

a) Using the dot product calculate the component of the vector $\underline{a}$ = (1, 0, 3) in the direction of the vector $\underline{b}$ = (-2, 3, 4).
b) Using the dot product calculate the angle between vector $\underline{a}$ = (1, 0, 3) and vector $\underline{b}$ = (-2, 3, 4).

Question 3

If $\underline{a}.\underline{b} = 4$, $\underline{a}.\underline{c} = 8.5$, $\underline{a}.\underline{d} = 7$, $\underline{c}.\underline{b} = 0.33$, $b.d = 6$, $\underline{c}.\underline{d} = 11$, then what do the following equal (numerical answers without working are not acceptable)?

a) $\underline{b}.\underline{c}$
b) $\underline{a}.(\underline{b}+\underline{c})$
c) $(\underline{d}+\underline{c}).(\underline{a}+\underline{b})$

Question 4

If $\underline{a}.\underline{b} = \underline{a}.\underline{c} = \underline{c}.\underline{b} = 0$ then write down vector $\underline{v}$ as a sum of $\underline{a}$, $\underline{b}$ and $\underline{c}$.

Question 5

a) A snooker player hits the cue ball towards another equal mass ball. After the collision, which you can assume conserves kinetic energy, the balls appear to be travelling at right angles. Prove this is always true using a dot product.

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